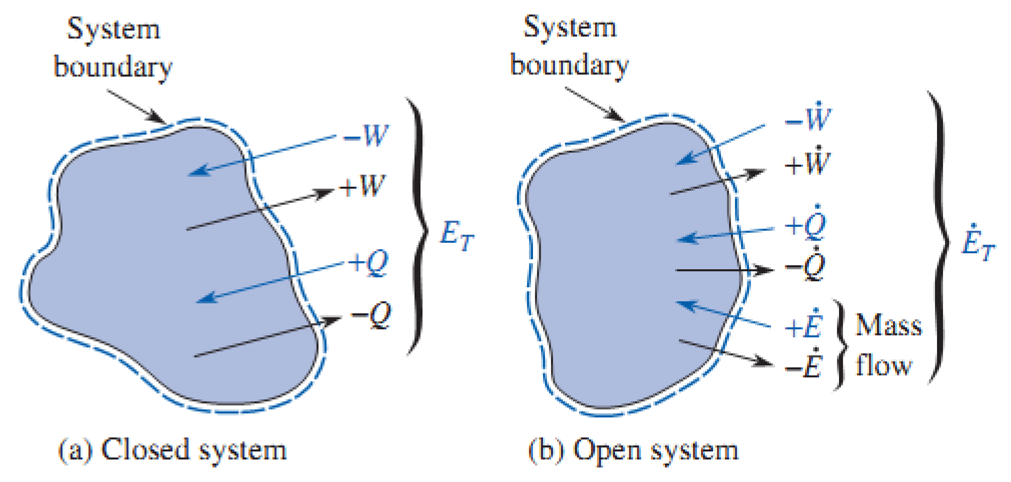
Chapter 6 Energy Equation for a Control Volume

Conservation of Mass and the Control Volume

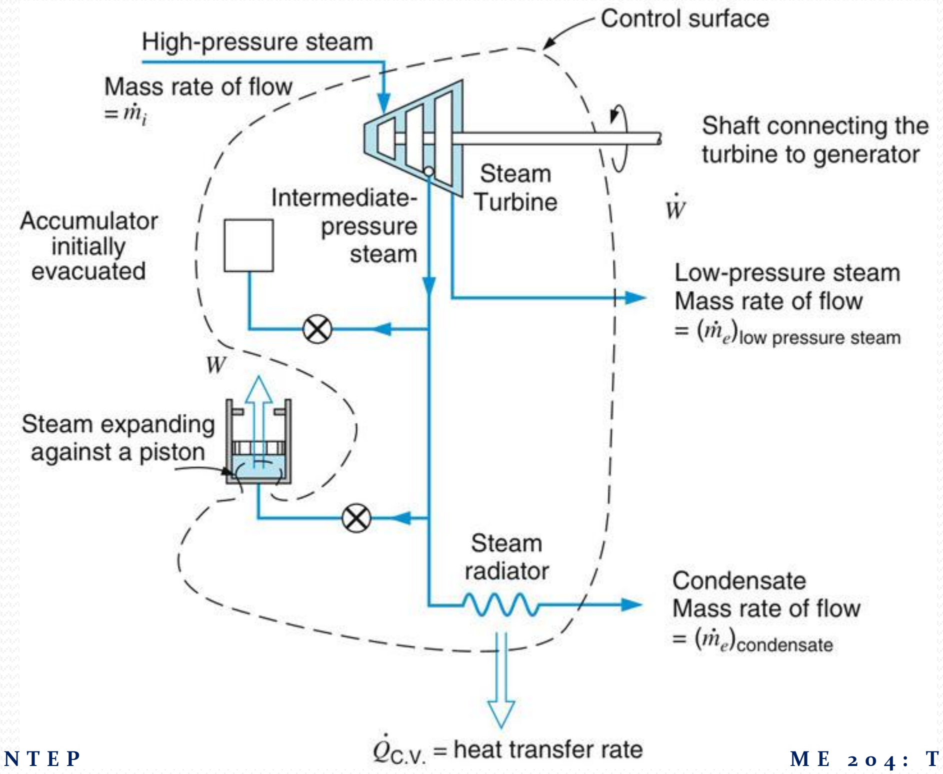
Closed systems: The mass of the system remain constant during a process.

Control volumes: Mass can cross the boundaries, and so we must keep track of the amount of mass entering and leaving the control volume.



(a) Closed system (b) Open system

A control volume is a volume in space in which one has interest for a particular study or analysis. The surface of this control volume is referred to as a control surface and always consists of a closed surface. The size and shape of the control volume are completely arbitrary and are so defined as to best suit the analysis to be made. The surface may be fixed, or it may move so that it expands or contracts.



Conservation of mass: Mass, like energy, is a conserved property, and it cannot be created or destroyed during a process.

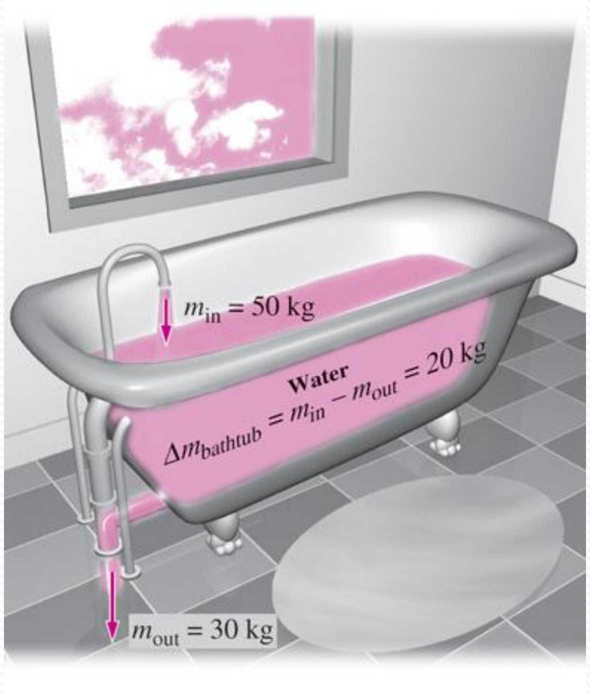
The rate of change of mass inside the control volume can be different from zero if we add or take a flow of mass out as

$$Rate\ of\ \ change = +in - out$$

$$\begin{bmatrix}\text{Sum of rate}\\ \text{of mass flowing}\\ \text{into control volume}\end{bmatrix} - \begin{bmatrix}\text{Sum of rate}\\ \text{of mass flowing}\\ \text{from control volume}\end{bmatrix} = \begin{bmatrix}\text{Time rate change}\\ \text{of mass inside}\\ \text{control volume}\end{bmatrix}$$

or

$$\frac{dm_{cv}}{dt} = \sum \dot{m}_i - \sum \dot{m}_e$$



Mass and Volume Flow Rates

$$\delta\dot{m} = \rho V_n \, dA_c$$

$$\dot{m} = \int_{A_c} \delta\dot{m} = \int_{A_c} \rho V_n \, dA_c$$

$$\dot{m} = \rho V_{\text{avg}} A_c \qquad (\text{kg/s})$$

$$\dot{m} = \rho\dot{V} = \frac{\dot{V}}{v}$$

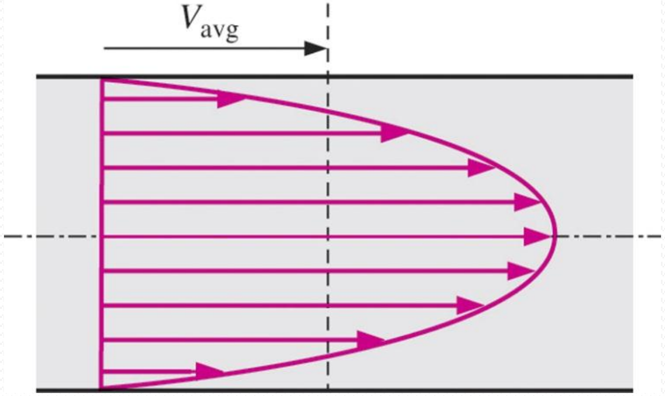
Mass flow rate

$$V_{\text{avg}} = \frac{1}{A_c} \int_{A_c} V_n \, dA_c$$

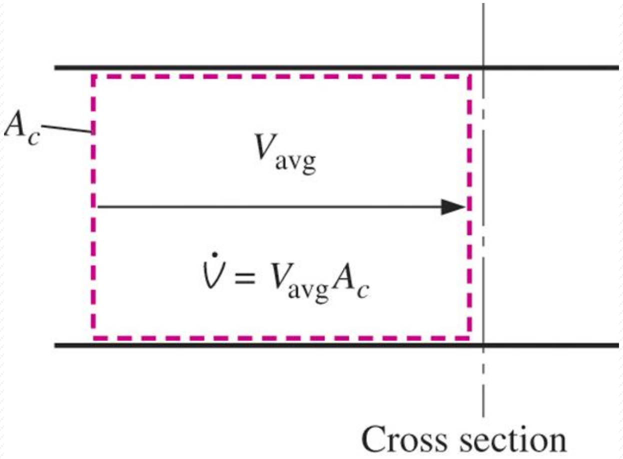
Definition of average velocity

Volume flow rate

$$\dot{V} = \int_{A_c} V_n \, dA_c = V_{\text{avg}} A_c = VA_c \qquad (\text{m}^3/\text{s})$$



The average velocity V_{avg} is defined as the average speed through a cross section.



The volume flow rate is the volume of fluid flowing through a cross section per unit time.

The First Law of Thermodynamics for a Control Volume

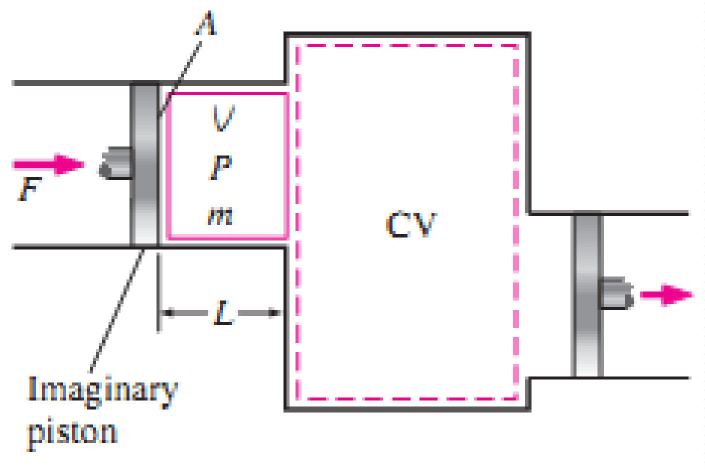
The first law of thermodynamics **for a control mass** $E_2 - E_1 = {}_1Q_2 - {}_1W_2$

Or time rate form $\dot{E}_{CM} = \dot{Q} - \dot{W}$

The fluid flowing across the control surface enters or leaves with an amount of energy per unit mass as

$$e = u + \frac{1}{2}V^2 + gZ$$

Whenever a fluid mass enters a control volume at state i, or exits at state e, there is a boundary movement work.



Some work is required to push the mass into or out of the control volume. This work is known as the **flow work, or flow energy**

$$W_{\text{flow}} = FL = PAL = PV$$

Total Energy of a Flowing Fluid

Total energy per unit mass associated with the flow of mass is

$$e + Pv = u + Pv + \frac{1}{2}V^2 + gZ = h + \frac{1}{2}V^2 + gZ$$

The extension of the first law of thermodynamics from $\dot{E}_{CM} = \dot{Q} - \dot{W}$

$$\frac{dE_{\text{C.V.}}}{dt} = \dot{Q}_{\text{C.V.}} - \dot{W}_{\text{C.V.}} + \dot{m}_i e_i - \dot{m}_e e_e + \dot{W}_{\text{flow in}} - \dot{W}_{\text{flow out}}$$

Or

$$\frac{dE_{\text{C.V.}}}{dt} = \dot{Q}_{\text{C.V.}} - \dot{W}_{\text{C.V.}} + \dot{m}_i(e_i + P_i v_i) - \dot{m}_e(e_e + P_e v_e)$$

Or

$$\dot{Q}_{cv} - \dot{W}_{cv} = \dot{E}_{cv} + \dot{m}_e(h + ke + pe)_e - \dot{m}_i(h + ke + pe)_i$$

For the general control volume we may have ***several entering or leaving mass flow rates***, so a summation over those terms is often needed. The final form of the first law of thermodynamics then becomes

$$\frac{dE_{\text{C.V.}}}{dt} = \dot{Q}_{\text{C.V.}} - \dot{W}_{\text{C.V.}} + \sum \dot{m}_i\left(h_i + \frac{1}{2}\mathbf{V}_i^2 + gZ_i\right) - \sum \dot{m}_e\left(h_e + \frac{1}{2}\mathbf{V}_e^2 + gZ_e\right)$$

As the kinetic and potential energy terms per unit mass appear together with the enthalpy in all the flow terms, **a shorter notation** is often used

$$h_{tot} = h + \frac{1}{2}V^2 + g\,Z \qquad\qquad h_{stag} = h + \frac{1}{2}V^2$$

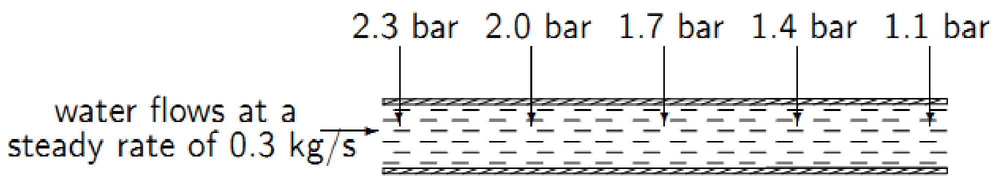
defining the **total enthalpy** and the **stagnation enthalpy**. The shorter equation then becomes

$$\frac{dE_{\text{C.V.}}}{dt} = \dot{Q}_{\text{C.V.}} - \dot{W}_{\text{C.V.}} + \sum \dot{m}_i h_{\text{tot},i} - \sum \dot{m}_e h_{\text{tot},e}$$

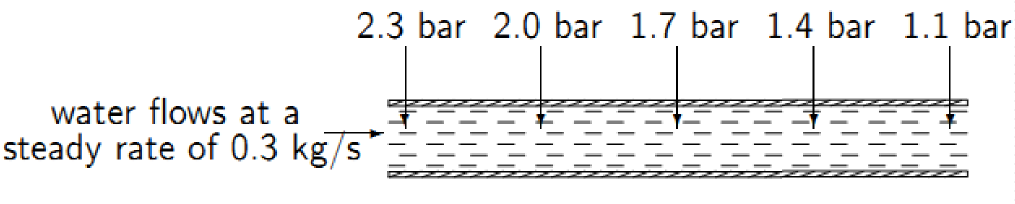
The Steady–State Process

The term **steady** implies **no change with time**. We say that a person is running at a steady speed of 5 km per hour, as shown in Figure, if the speed does not change with time.

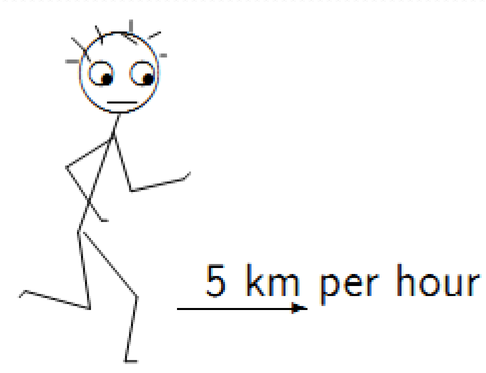
$$\frac{d(\ )_{CV}}{dt}=0$$



(a) measurements taken at 10.00 am



(b) measurements taken at 2.00 pm



A person running at a steady speed of 5 km per hour.

The properties of the flow is **not changed with time. However,** the properties need not be the same in all points of the flow

A steady-flow process is concerned primarily with the properties of the fluid entering and leaving the control volume.

A steady-state process is primarily interested in the spatial distribution of properties, particularly temperature.

Since this is an introductory book **we will use the term steady-state process for both**.

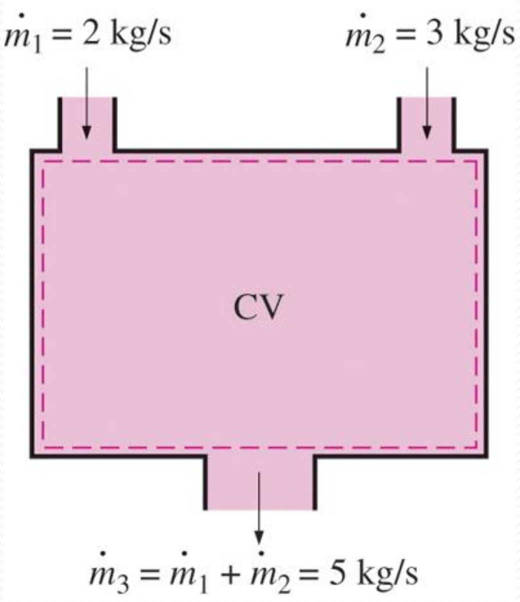
Consider a certain set of assumptions that lead to a reasonable model for this type of process, which we refer to as the **steady-state process**.

1. The control volume does not move relative to the coordinate frame.

2. The state of the mass at each point in the control volume does not vary with time.

So during a steady-flow process, the total amount of mass contained within a control volume does not change with time (m_{CV} = constant).

Then the conservation of mass principle requires that the total amount of mass entering a control volume equal the total amount of mass leaving it.



Conservation of mass principle for a two-inlet–one-outlet steady-flow system.

For steady-flow processes, we are interested in the amount of mass flowing per unit time, that is, *the mass flow rate.*

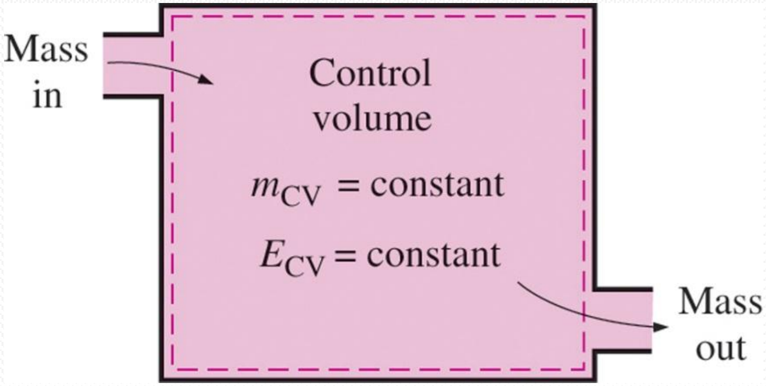
For multiple inlets and exits

$$\sum_{\text{in}} \dot{m} = \sum_{\text{out}} \dot{m} \qquad (\text{kg/s})$$

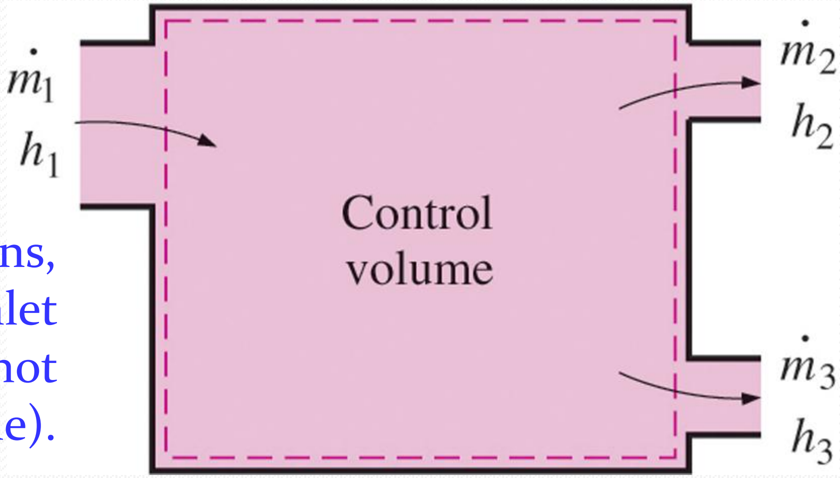
Since $$\frac{dm_{cv}}{dt} = 0$$

Energy Analysis of Steady-flow Systems

Many engineering systems such as power plants operate under steady conditions.



Under steady-flow conditions, the mass and energy contents of a control volume remain constant.



Under steady-flow conditions, the fluid properties at an inlet or exit remain constant (do not change with time).

The First Law of Thermodynamics for a Control Volume

$$\frac{dE_{\text{C.V.}}}{dt} = \dot{Q}_{\text{C.V.}} - \dot{W}_{\text{C.V.}} + \sum \dot{m}_i h_{\text{tot},i} - \sum \dot{m}_e h_{\text{tot},e}$$

The total energy content E of the control volume remain constant for a steady state process so

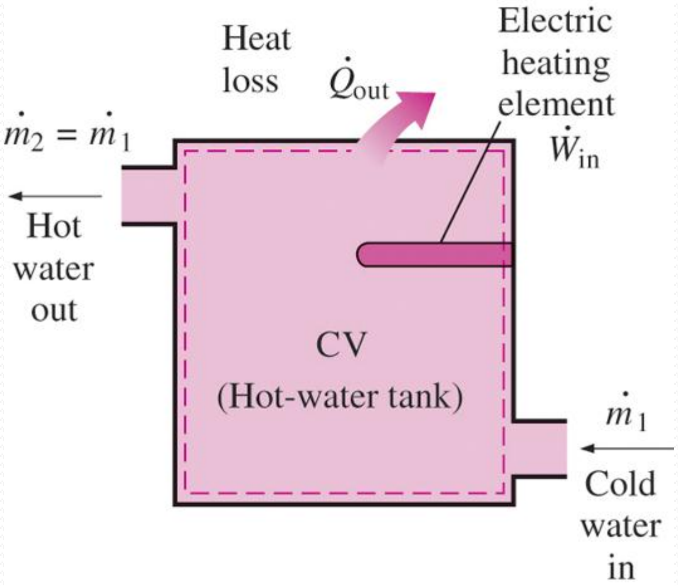
$$\frac{dE_{cv}}{dt} = 0$$

and the First Law of Thermodynamics for a steady state process

$$First\ Law: \quad \dot{Q}_{cv} - \dot{W}_{cv} = \sum \dot{m}_e h_{tot,e} - \sum \dot{m}_i h_{tot,i}$$

3. As for the mass that flows across the control surface, the mass flux and the state of this mass at each discrete area of flow on the control surface do not vary with time. The rates at which heat and work cross the control surface remain constant.

Many of the applications of the steady-state model are such that there is only **one flow stream entering and one leaving the control volume**. For this type of process



$$\dot{m}_1 = \dot{m}_2 \quad \rightarrow \quad \rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

Single stream

$$First\ Law: \quad \dot{Q}_{cv} - \dot{W}_{cv} = \dot{m}(h_{tot,e} - h_{tot,i})$$

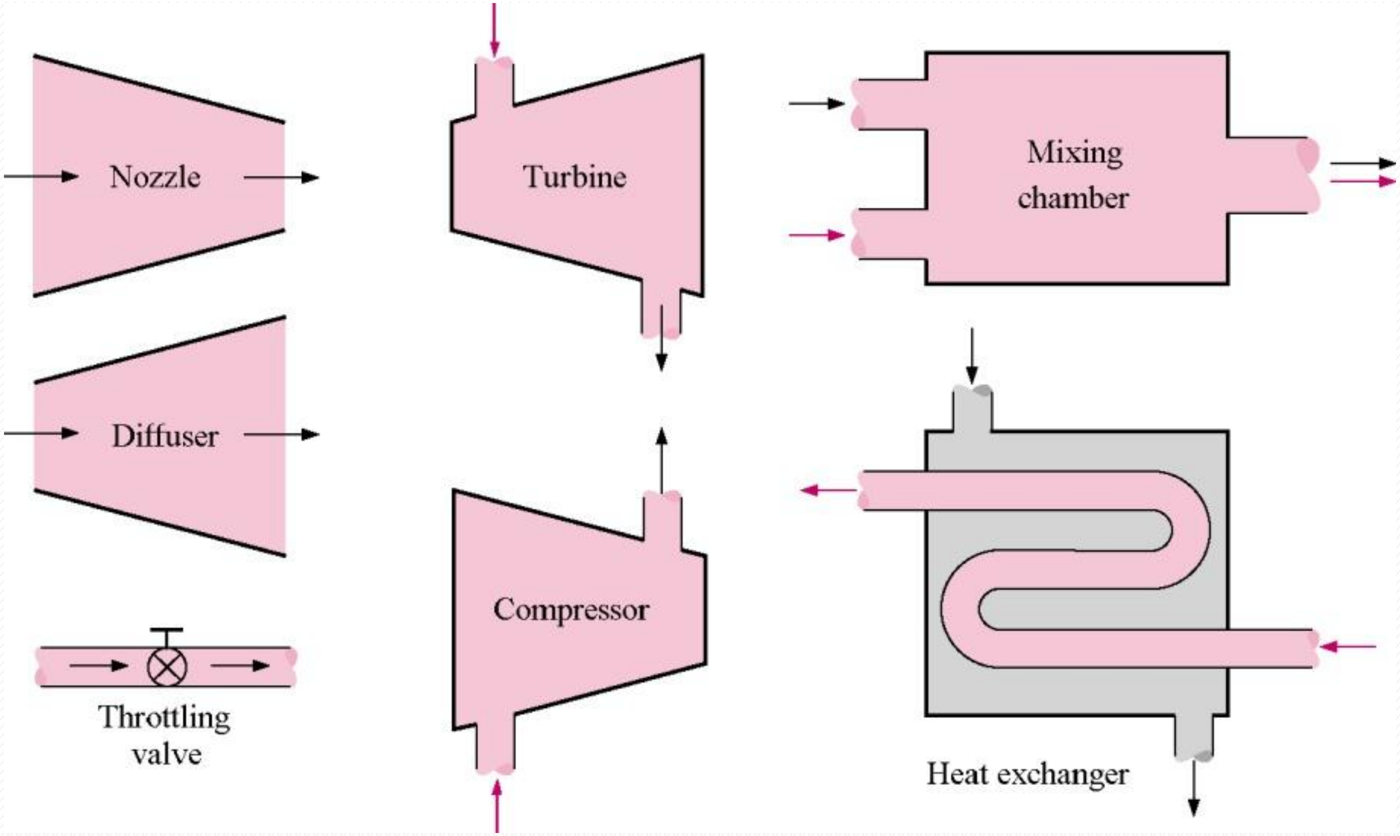
$$First\ Law: \quad q - w = \Delta\left(h_{tot}\right)_{ie}$$

where

$$q = \frac{\dot{Q}_{cv}}{\dot{m}} \qquad and \qquad w = \frac{\dot{W}_{cv}}{\dot{m}}$$

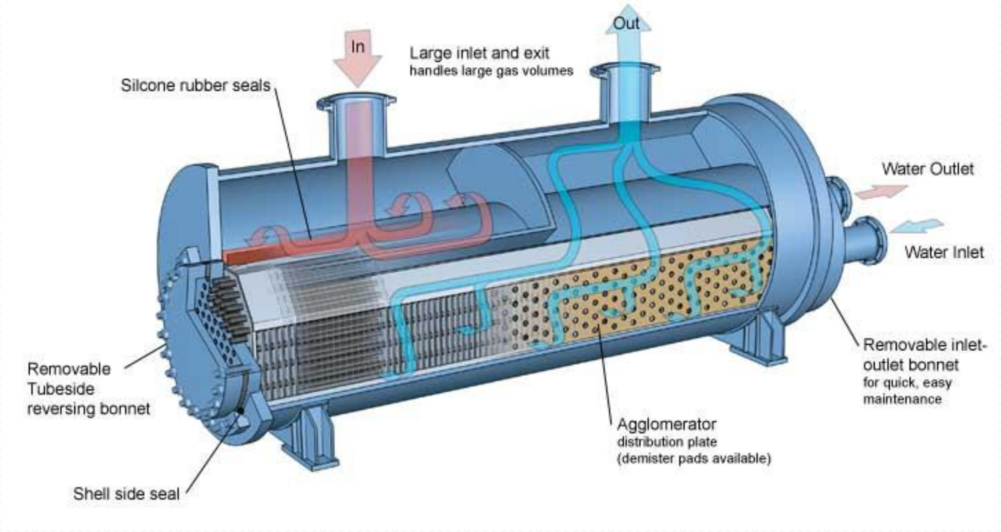
Examples of Steady–State Process

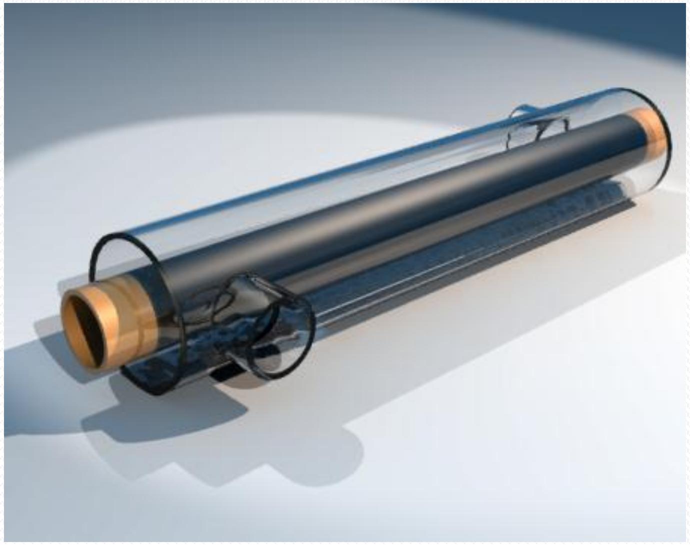
A number of examples of steady-state processes in which there is one fluid stream entering and one leaving the control volume or more than one fluid stream.



Heat Exchanger

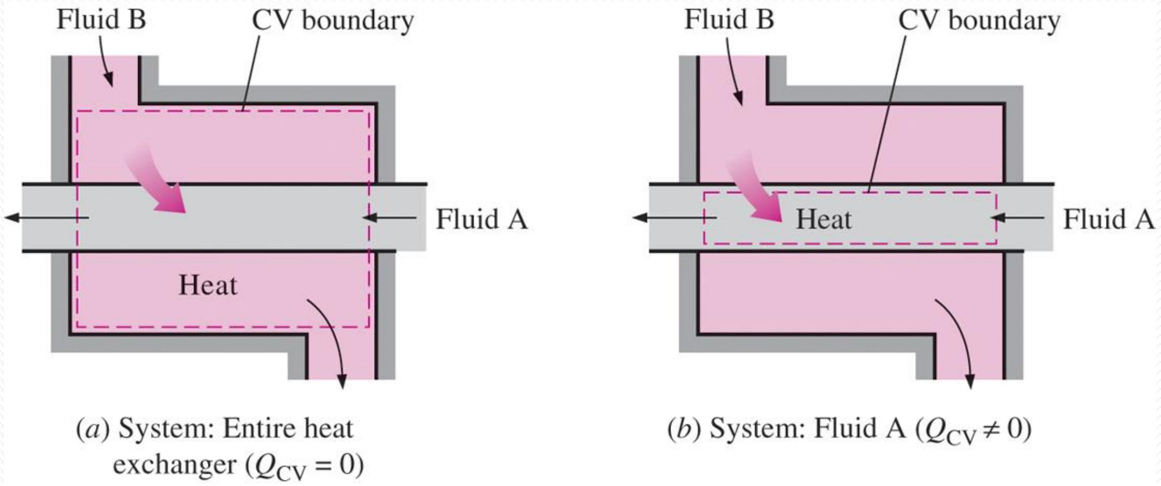
A steady-state heat exchanger is a simple fluid flow through a pipe or system of pipes, where heat is transferred to or from the fluid. The fluid may be heated or cooled, and may or may not boil, liquid to vapor, or condense, vapor to liquid.





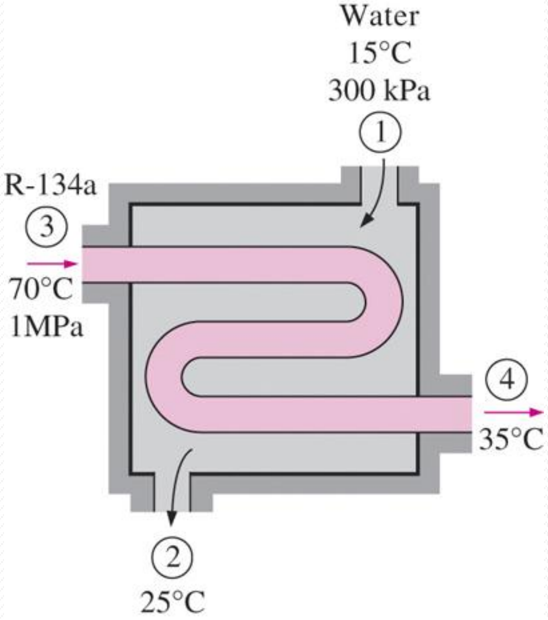
Tubular heat exchanger.

A heat exchanger can be as simple as two concentric pipes. The heat transfer associated with a heat exchanger may be zero or nonzero depending on how the control volume is selected.



(a) System: Entire heat exchanger ($Q_{CV} = 0$)

(b) System: Fluid A ($Q_{CV} \neq 0$)



Mass and energy balances for the adiabatic heat exchanger in the figure is:

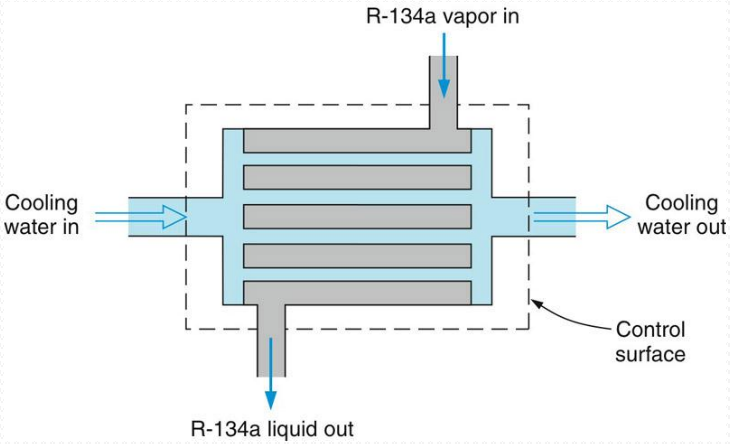
$$\dot{m}_1 = \dot{m}_2 = \dot{m}_w$$

$$\dot{m}_3 = \dot{m}_4 = \dot{m}_R$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m}_1 h_1 + \dot{m}_3 h_3 = \dot{m}_2 h_2 + \dot{m}_4 h_4$$

EXAMPLE 6.3 Consider a water-cooled condenser in a large refrigeration system in which R-134a is the refrigerant fluid. The refrigerant enters the condenser at 1.0 MPa and 60°C, at the rate of 0.2 kg/s, and exits as a liquid at 0.95 MPa and 35°C. Cooling water enters the condenser at 10°C and exits at 20°C. Determine the rate at which cooling water flows through the condenser.



It is reasonable to assume that both kinetic and potential energy changes are negligible. We note that the work is zero, and we make the other reasonable assumption that there is no heat transfer across the control surface. Therefore, the first law,

$$\sum \dot{m}_i h_i = \sum \dot{m}_e h_e$$

Using the subscript r for refrigerant and w for water, we write

$$\dot{m}_r (h_i)_r + \dot{m}_w (h_i)_w = \dot{m}_r (h_e)_r + \dot{m}_w (h_e)_w$$

by considering entire heat exchanger as a control volumes

From the R-134a and steam tables, we have $(h_i)_r = 441.89$ kJ/kg, $(h_i)_w = 42.00$ kJ/kg
$(h_e)_r = 249.10$ kJ/kg, $(h_i)_w = 83.95$ kJ/kg

Solving the above equation for $\dot{m}_w$ **the rate of flow of water**, we obtain

$$\dot{m}_w = \dot{m}_r \frac{(h_i - h_e)_r}{(h_e - h_i)_w} = 0.2\frac{441.89 - 241.10}{83.95 - 42.00} = 0.919\,kg/s$$

by considering two separate control volumes, one having the flow of R-134a across its control surface and the other having the flow of water across its control surface

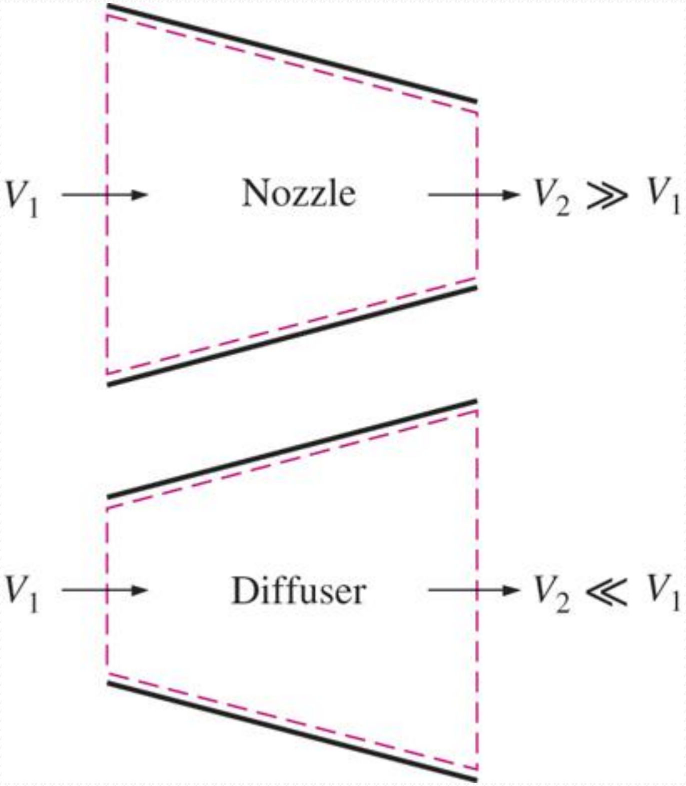
The heat transfer for the control volume involving R-134a is calculated first

$$\dot{Q}_{cv} = \dot{m}_r (h_e - h_i)_r = 0.2 * (249.10 - 441.89) = -38.56\,kW$$

This is also the heat transfer to the other control volume, for which

$$\dot{Q}_{cv} = +38.56\,kW \qquad \dot{Q}_{cv} = \dot{m}_w (h_e - h_i)_w \qquad \dot{m}_w = \frac{38.56}{83.95 - 42.00} = 0.919\,kg/s$$

Nozzles and Diffusers



Nozzles and diffusers are shaped so that they cause large changes in fluid velocities and thus kinetic energies.

A nozzle is a steady-state device whose purpose is to create a high-velocity fluid stream at the expense of the fluid's pressure.

A steady – state diffuser is a device constructed to decelerate a high – velocity fluid in a manner that results in an increase in pressure of the fluid. In essence, it is the exact opposite of a nozzle.

Nozzles

There is no means to do any work - there are no moving parts. There is little or no change in potential energy and usually little or no heat transfer.

Diffusers

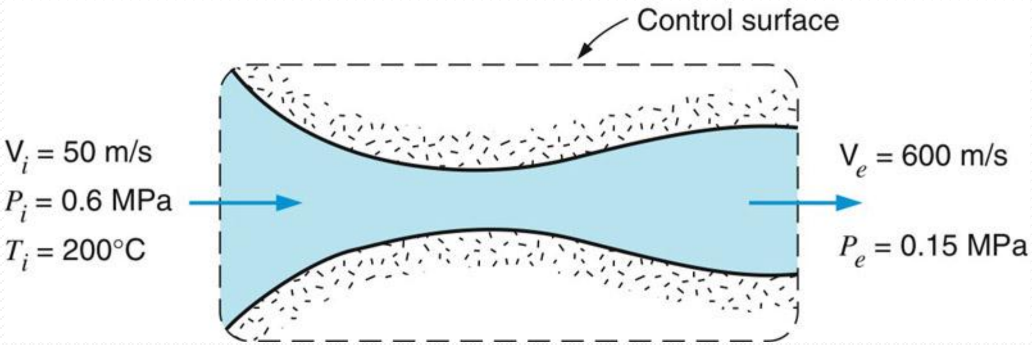
The assumptions are similar to those for a nozzle, with a large kinetic energy at the diffuser inlet and a small, but usually not negligible, kinetic energy at the exit being the only terms besides the enthalpies remaining in the first law.

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{m}\left(h_2 + \frac{V_2^2}{2}\right)$$

$$(\text{since } \dot{Q} \cong 0,\ \dot{W} = 0, \text{ and } \Delta\text{pe} \cong 0)$$

EXAMPLE 6.4 Steam at 0.6 MPa and 200°C enters an insulated nozzle with a velocity of 50 m/s. It leaves at a pressure of 0.15 MPa and a velocity of 600 m/s. Determine the final temperature if the steam is superheated in the final state and the quality if it is saturated.



$$\dot{W}_{cv} = 0 \qquad \dot{Q}_{cv} = 0$$

The first law yields

$$h_i + \frac{V_i^2}{2} = h_e + \frac{V_e^2}{2}$$

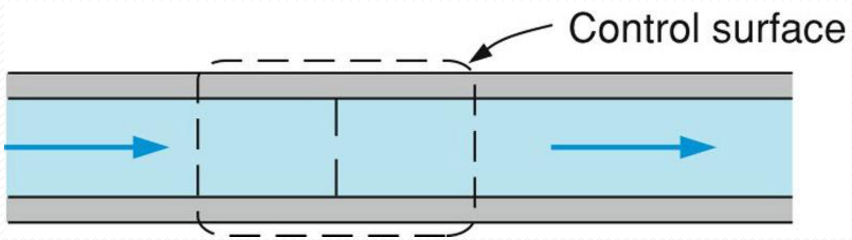
$$h_e = 2850.1 + \left(\frac{50^2}{2*1000} - \frac{600^2}{2*1000} \right) = 2671.4\, kJ/kg$$

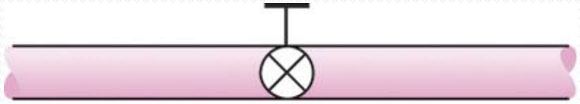
The two properties of the fluid leaving that we now know are pressure and enthalpy, and therefore the state of this fluid is determined. Since h_e is less than h_g and greater than at h_f 0.15 MPa (Sat. Temp. 111°C), the quality is calculated.

$$h = h_f + x h_{fg} \qquad 2671.4 = 467.1 + x_e * 2226.5 \qquad x_e = 0.99$$

Throttle

A throttling process occurs when a fluid flowing in a line suddenly encounters a restriction in the flow passage. This may be a plate with a small hole in it, as shown in Figure. it may be a partially closed valve protruding into the flow passage, or it may be a change to a tube of much smaller diameter, called a capillary tube.




(*a*) An adjustable valve


(*b*) A porous plug


(*c*) A capillary tube

The pressure drop in the fluid is often accompanied by a *large drop in temperature*, and for that reason throttling devices are commonly used in refrigeration and air-conditioning applications.

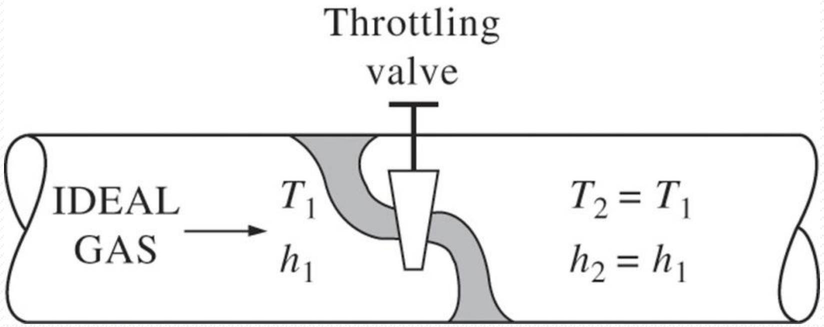
Consider fluid flowing through a one-entrance, one-exit porous plug. The fluid experiences a pressure drop as it flows through the plug. No net work is done by the fluid. Assume the process is adiabatic and that the kinetic and potential energies are neglected; then the conservation of mass and energy equations become

$$\dot{m}_i = \dot{m}_e$$

$$\dot{Q}_{net} + \underbrace{\sum \dot{m}_i \left(h_i + \frac{\vec{V}_i^{\,2}}{2} + gz_i \right)}_{\text{for each inlet}} = \dot{W}_{net} + \underbrace{\sum \dot{m}_e \left(h_e + \frac{\vec{V}_e^{\,2}}{2} + gz_e \right)}_{\text{for each exit}}$$

$$\dot{m}_i h_i = \dot{m}_e h_e$$

$$h_i = h_e$$



The temperature of an ideal gas does not change during a throttling (*h* = constant) process since *h* = *h*(*T*).

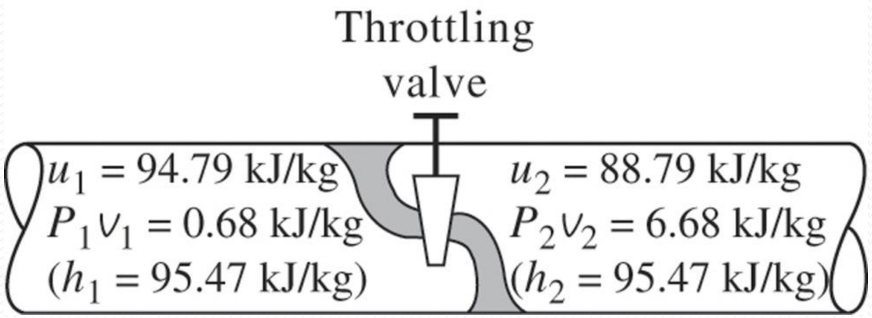
$$h_i = h_e$$

$$h_e - h_i = 0$$

$$\int_i^e C_p(T)\,dT = 0$$

$$or$$

$$T_e = T_i$$



During a throttling process, the enthalpy of a fluid remains constant. But internal and flow energies may be converted to each other.

EXAMPLE 6.5 Consider the following process, in which ammonia is the refrigerant. The ammonia enters the expansion valve at a pressure of 1.50 MPa and a temperature of 35°C, Its pressure on leaving the expansion valve is 291 kPa. Calculate the quality of the ammonia leaving the expansion valve.

We can use standard throttling process analysis and assumptions. The first law reduces to

$$h_i = h_e$$

From the ammonia tables we get

$$h_i = 346.8 \text{ kJ/kg}$$

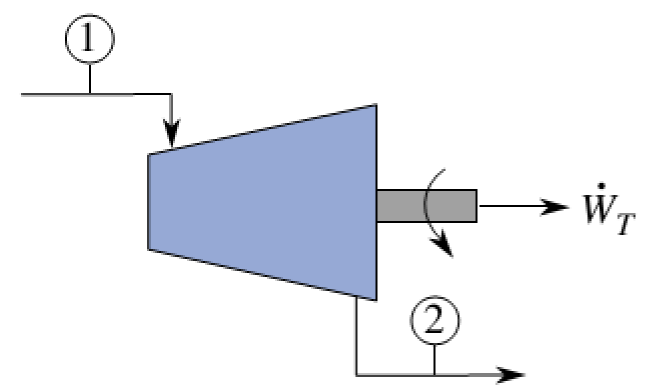
(The enthalpy of a slightly compressed liquid is essentially equal to the enthalpy of saturated liquid at the same temperature.)

$$h_e = h_i = 346.8 = 134.4 + x_e * 1296.4$$

$$x_e = 0.1638 = 16.38\%$$

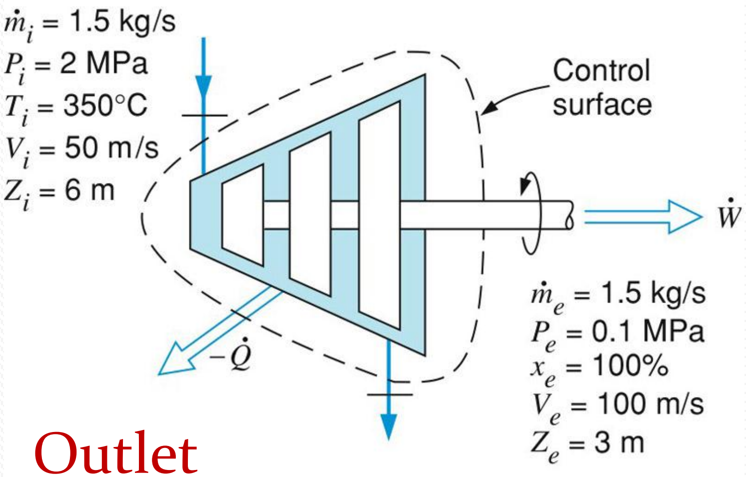
Turbine

A turbine is a rotary steady-state machine whose purpose is to produce shaft work at the expense of the pressure of the working fluid. Two general classes of turbines are steam turbines, in which the steam exiting the turbine passes to a condenser, where it is condensed to liquid, and gas turbines, in which the gas usually exhausts to the atmosphere from the turbine.



$$\dot{Q}_{cv} - \dot{W}_{cv} = \dot{m}(h_{tot,e} - h_{tot,i})$$

EXAMPLE 6.6 The mass rate of flow into a steam turbine is 1.5 kg/s, and the heat transfer from the turbine is 8.5 kW. Determine the power output of the turbine.



$$\dot{Q}_{cv} - \dot{W}_{cv} = \dot{m}(h_{tot,e} - h_{tot,i}) \qquad \dot{Q}_{cv} = -8.5\,kW$$

Inlet

Outlet

From the steam tables, $h_i = 3137.0$ kJ/kg

$$\frac{V_i^2}{2} = \frac{50^2}{2*1000} = 1.25\,kJ/kg$$

$$g\,Z_i = \frac{6*9.8066}{1000} = 0.059\,kJ/kg$$

Similarly, for the exit $h_e = 2675.5$ kJ/kg and

$$\frac{V_e^2}{2} = \frac{100^2}{2*1000} = 5.0\,kJ/kg$$

$$g\,Z_e = \frac{3*9.8066}{1000} = 0.029\,kJ/kg$$

Therefore, substituting into first law, we obtain

$$\dot{Q}_{cv} - \dot{W}_{cv} = \dot{m}(h_{tot,e} - h_{tot,i})$$

$$\dot{Q}_{cv} - \dot{W}_{cv} = \dot{m}[(h + \frac{1}{2}V^2 + gZ)_e - (h + \frac{1}{2}V^2 + gZ)_i]$$

$$-8.5 - \dot{W}_{cv} = 1.5 * [(2675.5 + 5.0 + 0.029) - (3137.0 + 1.25 + 0.059)]$$

$$\dot{W}_{cv} = 678.2\,kW$$

If heat loss neglected: $\dot{W}_{cv} = 686.67\,kW \quad +1.25\%$

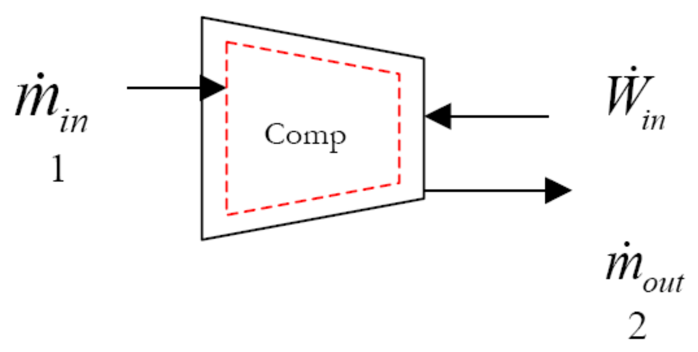
If ke change neglected: $\dot{W}_{cv} = 683.795\,kW \quad +0.83\%$

If pe change neglected: $\dot{W}_{cv} = 678.125\,kW \quad -0.007\%$

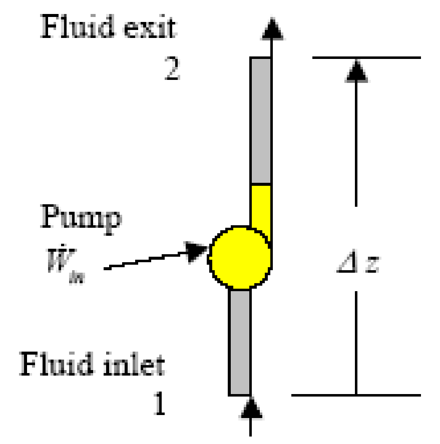
If all neglected (except Δh): $\dot{W}_{cv} = 692.25\,kW \quad +2.08\%$

Compressor and Pump

The purpose of a steady-state compressor (gas) or pump (liquid) is the same: to increase the pressure of a fluid by putting in shaft work



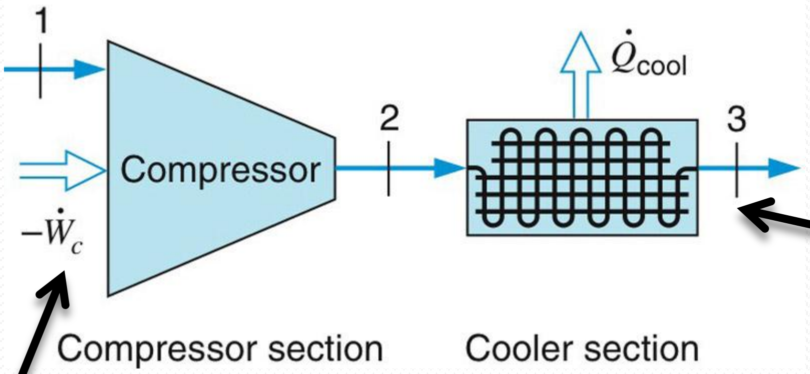
Steady-Flow Compressor



Liquid flow through a pump

$$\dot{Q} - \dot{W} = \dot{m}\left[h_2 - h_1 + \frac{\vec{V}_2^2 - \vec{V}_1^2}{2} + g(z_2 - z_1) \right] \qquad (kW)$$

EXAMPLE 6.7 The compressor in a plant receives carbon dioxide at 100 kPa, 280 K, with a low velocity. At the compressor discharge, the carbon dioxide exits at 1100 kPa, 500 K, with velocity of 25 m/s and then flows into a constant-pressure aftercooler (heat exchanger) where it is cooled down to 350 K. The power input to the compressor is 50 kW. Determine the heat transfer rate in the aftercooler.



$$q = h_3 - h_2 = 257.9 - 401.5 = -143.6\,kJ/kg$$

$$\dot{Q}_{cool} = -\dot{m}\,q = 0.245 * 143.6 = 35.2\,kW$$

Here we assume q = 0 and V_1 = 0, so, getting h from Table

$$q - w = \Delta\left(h_{tot}\right)_{ie} \qquad -w = h_2 - h_1 + \frac{1}{2}V_2^2 = 401.52 - 198 + \frac{25^2}{2000} = 203.8\,kJ/kg$$

$$\dot{m} = \frac{\dot{W}_c}{w} = \frac{-50}{-203.8} = 0.245\,kg/s$$

The Transient Process

Many processes of interest in thermodynamics involve unsteady flow. For example, filling of closed tanks or discharge from closed. We call this process **the transient process**, for convenience, recognizing that our model includes specific assumptions that are not always valid.

1. The control volume remains constant relative to the coordinate frame.

2. The state of the mass within the control volume may change with time, but at any instant of time the state is uniform throughout the entire control volume (or over several identifiable regions that make up the entire control volume).

3. The state of the mass crossing each of the areas of flow on the control surface is constant with time although the mass flow rates may be time varying.

4. The rates at which heat and work cross the control surface remain constant.

The overall process occurs during time t. At any instant of time during process, the continuity equation is

$$\frac{dm_{cv}}{dt} + \sum \dot{m}_e - \sum \dot{m}_i = 0$$

where the summation is over all areas on the control surface through which flow occurs. Integrating over time t gives the change of mass in the control volume during the overall process

$$\int \frac{dm_{cv}}{dt} dt = (m_2 - m_1)_{cv}$$

The total mass leaving the control volume during time t is

$$\int_0^t \sum \dot{m}_e dt = \sum m_e$$

and the total mass entering the control volume during time is

$$\int_0^t \sum \dot{m}_i dt = \sum m_i$$

Therefore, for this period of time t, we can write the continuity equation for the transient process as

$$(m_2 - m_1)_{cv} + \sum m_e - \sum m_i = 0$$

In writing the first law of the transient process at any instant of time during the process:

$$\dot{Q}_{cv} - \dot{W}_{cv} = \dot{E}_{cv} + \sum \dot{m}_e h_{tot,e} - \sum \dot{m}_i h_{tot,i}$$

Since at any instant of time the state within the control volume is uniform, the first law for the transient process becomes

$$\dot{Q}_{cv} - \dot{W}_{cv} = \frac{d}{dt}\left[m\left(u + \frac{V^2}{2} + gZ \right) \right]_{cv} + \sum \dot{m}_e h_{tot,e} - \sum \dot{m}_i h_{tot,i}$$

Let us now integrate this equation over time t, during which time we have

$$\int_0^t \dot{Q}_{cv}\, dt = Q_{cv} \qquad \int_0^t \dot{W}_{cv}\, dt = W_{cv}$$

$$\int_0^t \sum \dot{m}_i h_{tot,i}\, dt = \sum m_i \left(h + \frac{V^2}{2} + gZ \right)_i \qquad \int_0^t \sum \dot{m}_e h_{tot,e}\, dt = \sum m_e \left(h + \frac{V^2}{2} + gZ \right)_e$$

$$\int_0^t \left[m\left(u + \frac{V^2}{2} + gZ \right) \right]_{cv} dt =$$

$$\left[m_2 \left(u + \frac{V^2}{2} + gZ \right)_2 - m_1 \left(u + \frac{V^2}{2} + gZ \right)_1 \right]_{cv}$$

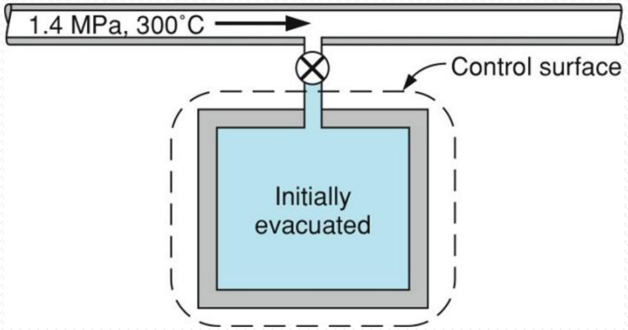
Therefore, for this period of time t, we can write the first law for the transient process as

$$Q_{cv} - W_{cv} = \left[m_2 \left(u + \frac{V^2}{2} + g\,Z \right)_2 - m_1 \left(u + \frac{V^2}{2} + g\,Z \right)_1 \right]_{cv}$$

$$+ \sum m_e \left(h + \frac{V^2}{2} + g\,Z \right)_e - \sum m_i \left(h + \frac{V^2}{2} + g\,Z \right)_i$$

As an example of the type of problem for which these assumptions are valid and above equation is appropriate

EXAMPLE 6.11 Steam at a pressure of 1.4 MPa and temperature of 300°C is flowing in a pipe (Figure 6.14). Connected to this pipe through a valve is an evacuated tank. The valve is opened and the tank fills with steam until the pressure is 1.4 MPa, and then the valve is closed. The process takes place adiabatically and kinetic energies and potential energies are negligible. Determine the final temperature of the steam.



$$Q_{cv} - W_{cv} = \left[m_2\left(u + \frac{V^2}{2} + gZ\right)_2 - m_1\left(u + \frac{V^2}{2} + gZ\right)_1 \right]_{cv} + \sum m_e\left(h + \frac{V^2}{2} + gZ\right)_e - \sum m_i\left(h + \frac{V^2}{2} + gZ\right)_i$$

We note that $Q_{cv} = 0$ $W_{cv} = 0$, $m_e = 0$, and $(m_1)_{c.v.} = 0$. We further assume that changes in kinetic and potential energy are negligible. Therefore, the statement of the first law for this process reduces to $m_i h_i = m_2 u_2$

From the continuity equation for this process $m_i = m_2$

Therefore, combining the continuity equation with the first law, $h_i = u_2$

From the steam tables we obtain $h_i = u_2 = 3040.4$ kJ/kg

Since the final pressure is given as 1.4 MPa, we know two properties at the final state and therefore the final state is determined. The temperature corresponding to a pressure of 1.4 MPa and an internal energy of 3040.4 kJ/kg is found to be 452°C.